Table 3. *Bond lengths and angles*

Cl–O_1	1·453 ± 0·009 Å	weighted mean 1·452 ± 0·005 Å
Cl–O_2	1·439 ± 0·009	
Cl–O_3	1·465 ± 0·008	

O_1–Cl–O_3	108° 9′ ± 30′
O_1–Cl–O_2	111° 30′ ± 30′
O_2–Cl–O_3	110° 30′ ± 30′

again demonstrated the importance of correcting for rotational oscillation.

Hydrogen atoms

Lee & Carpenter were unable to find the hydrogen atoms in projection, nor did they find three $O_5 \cdots O$ distances shorter than the others such as might be expected for a hydrogen-bonded H_3O^+ ion. They suggested either (i) that there is free rotation of the hydrogen atoms about the oxygen atom, or (ii) that the hydrogen atoms were equidistant from pairs of oxygen atoms on the neighbouring perchlorate ions.

In an endeavour to locate the hydrogen atoms a three-dimensional difference synthesis was carried out with F_c from the final anisotropic parameters. As might be expected from the small value of R, there were no large peaks or throughs and the positive values within a radius of 1·5 Å from O_5 were smaller than those in positions which could not be attributed to hydrogen atoms. This evidence indicates the free rotation of the hydroxonium ions, and is consistent with the preliminary results of a neutron-diffraction study (Smith & Levy, 1959).

Computation was carried out on the Leeds University Ferranti Pegasus computer with programmes published in 1960 (Cruickshank, Pilling and in part Bujosa, Lovell & Truter).

I am grateful for the use of the facilities of the Computing Laboratory and to my colleagues for the use of their programmes.

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Coloured polyhedra.

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Introduction

Klein (1884) gave an account of the subgroups contained in any point-group, but thought it unnecessary to enumerate these subgroups in his book. Recently Wittke & Garrido (1959) tabulated some of these, but restricted their attention to crystallographic point-groups. In this present note the treatment is extended to the icosahedral point-groups, and the general case for the point-groups r, $r2$, rm, $\bar{r}$, $\bar{r}m$, r/m and r/mmm can be obtained by Klein's analysis. The notation here used is the same as that of Wittke & Garrido.

Definition of a coloured polyhedron

A 'coloured polyhedron' has its shape governed by a point-group **G** but has its faces coloured in a way that obeys a different point-group **g**. **g** is a subgroup of **G**, and **g**′ is any group for which **g**.**g**′ = **G**. **G** and **g** have M and m_g faces in their general forms respectively, thus the numner of colours s required for the general coloured polyhedron is M/m_g. All the symmetry operations of **G** not included in **g** do not necessarily form a group, but must change the colour of any face on which they act, and these are called colour symmetry operations.

Multicoloured faces

The number of different colours related by m_c (colour mirror), r_c (colour r-fold axis) and r_cm_c (r_c in the plane of m_c) are clearly 2, r and $2r$ respectively. A special form of **G** is produced if the face normal lies in a mirror plane or parallels an r-fold axis, and this is denoted in the tables by m or r. The faces of any special form can be sectioned by lines in such a way that the sections, considered as separate, obey the general point-group symmetry. These lines can be obtained from the edges

Table 1. *The number of colours n present on each face of the special forms of the point group 5m3m*

g	m_g	s	5	3	2	m	**g**′	*
$5m3m$	120	1	1	1	1	1	1	
532	60	2	2	2	2	2	m	
52	10	12	2,10	6	2,4	2	$3m$	
5	5	24	2,10	6	4	2	$3m$	
$\bar{5}$	10	12	2,10	6	4	2	$3m$	
$5m$	10	12	1,5	3	2	1,2	3	
$\bar{5}m(2)$	20	6	1,5	3	2	1,2	3	
$m3(2)$	24	5	5	2,3	1,4	1,2	5	*
23	12	10	10	2,6	2,4	2	$5m$	*
32	6	20	10	2,6	2,4	2	$5m$	*
3	3	40	10	2,6	4	2	$5m$	*
$\bar{3}$	6	20	10	2,6	4	2	$5m$	*
$3m$	6	20	5	1,6	2,4	1,2	5	*
$\bar{3}m(2)$	12	10	5	1,6	2,4	1,2	5	*
222	4	30	10	6	2,4	2	$5m$	*
2	2	60	10	6	2,4	2	$5m3m$	*
$mmm(2)$	8	15	5	3,6	1,4	1,2	5	*
$mm(2)$	4	30	5,10	3,6	1,2,4	1,2	532	*
$2/m$	4	30	5,10	3,6	2,4	1,2	532	*
m	2	60	5,10	3,6	2,4	1,2	532	*
$\bar{1}$	2	60	10	6	4	2	532	*
1	1	120	10	6	4	2	$5m3m$	*

* Denotes a crystallographic subgroup.

Table 2. *The number of colours n present on each face of the special forms of the point group* 532

$\mathfrak{g}$	m_g	s	5	3	2	$\mathfrak{g}'$	*
532	60	1	1	1	1	1	
52	10	6	1,5	3	1,2	3	
5	5	12	1,5	3	2	3	
23	12	5	5	1,3	1,2	5	*
32	6	10	5	1,3	1,2	5	*
3	3	20	5	1,3	2	5	*
222	4	15	5	3	1,2	5	*
2	2	30	5	3	1,2	52	*
1	1	60	5	3	2	532	*

* Denotes a crystallographic subgroup.

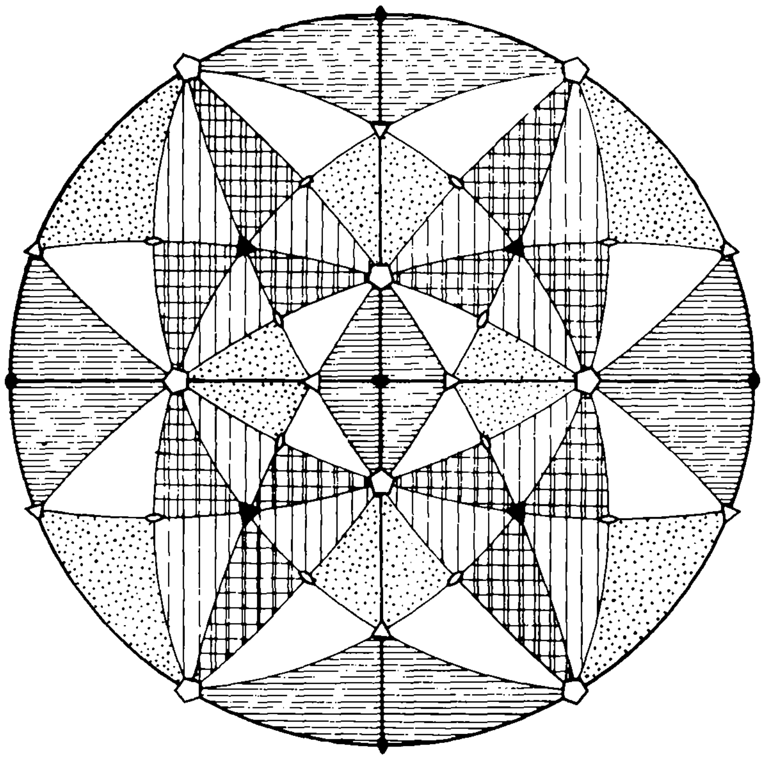

Fig. 1. $\mathbf{G} = 5m3m$, $\mathfrak{g} = m3$.

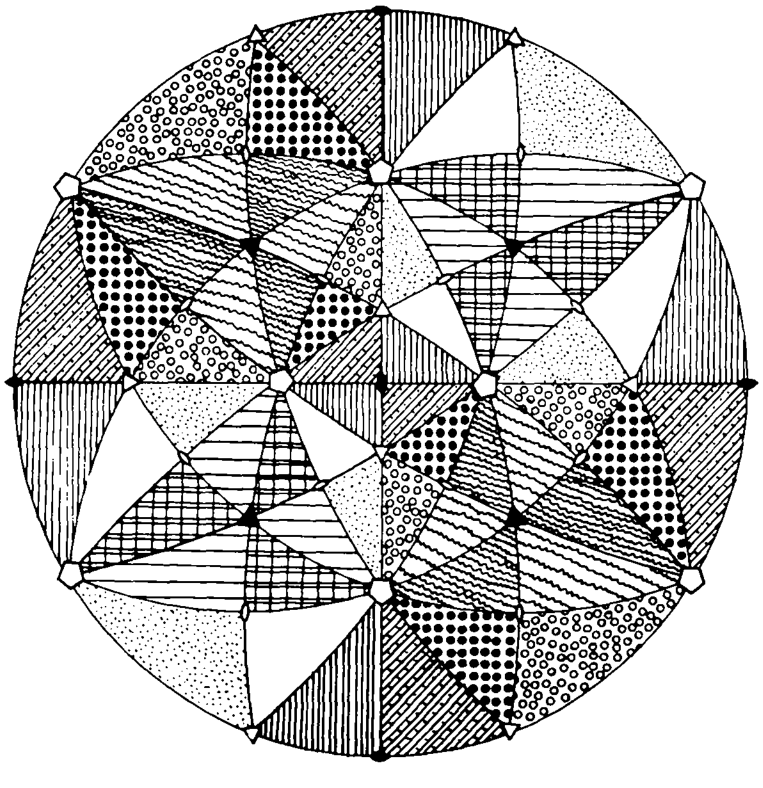

Fig. 2. $\mathbf{G} = 5m3m$, $\mathfrak{g} = 23$.

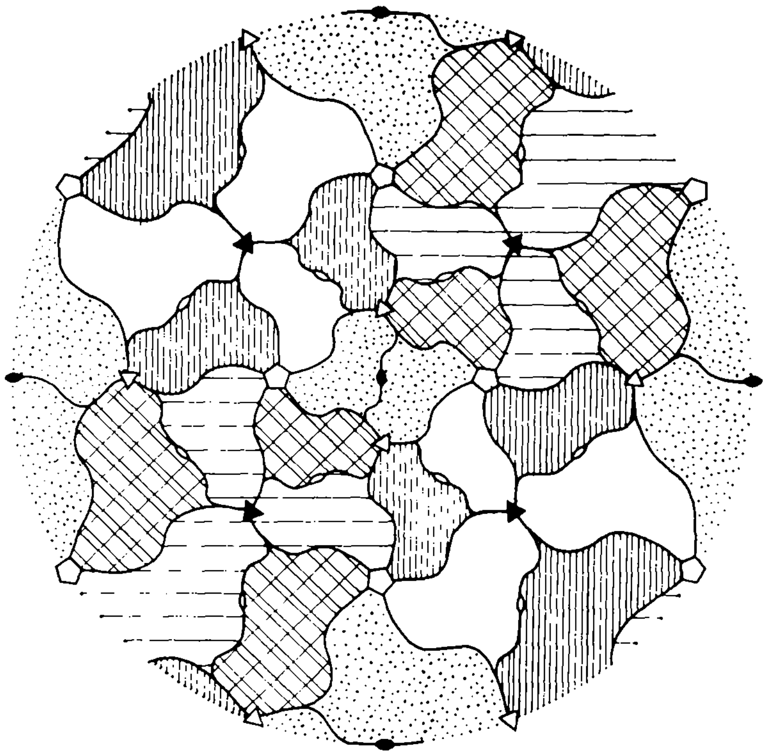

Fig. 3. $\mathbf{G} = 532$, $\mathfrak{g} = 23$.

between faces of the general form whose normals tend to parallel those of the special form. If the special form face normal lies in an m_c, or is parallel to an r_c, or both, then the sections of this face will be in n different colours, which will be 2, r and $2r$ respectively. Wittke & Garrido superimposed all these n colours on the face, but then this sometimes destroys colour operators. A simple example is given by $\mathbf{G} = m3m$ and $\mathfrak{g} = 23$ with the special form (110), (or in this notation 2). All the faces then have $n = 4$ and if these are superimposed the colour mirror planes become true mirrors.

A symmetry operator, say r, might occur more than once in $\mathbf{G}$ but not all of these need belong to $\mathfrak{g}$. Then some faces of the special form produced when the face normals are parallel to the r-fold axis will be brought into self-coincidence by an r_c, whereas the other faces will still have $n = 1$. When this occurs the two values of n are tabulated in the same space.

Tables 1 and 2 give n for the icosahedral point-groups, and a note is made of the cases where $\mathfrak{g}$ is a crystallographic point-group. The interesting cases, Figs. 1, 2 and 3 have cubic subgroups. Their importance lies in the fact that some viruses which crystallize in the cubic system have the symmetry of the point-group $5m3m$ or 532 about each lattice point (Horne *et al.*, 1959).

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